New Results in Polyphase Sampling

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The famous Whittaker cardinal series, or Shannon's sampling theorem, tells that band-limited functions can be written in terms of the integer-translates of the sinc function $\phi(x) := \frac{\sin \pi x}{\pi x}$ as

$$f(x) = \sum_{\alpha \in \mathbf{Z}} f(\alpha)\ \phi(x - \alpha)$$

if the support of the Fourier transform $f^\wedge(\xi) := \int_{\mathbf{R}} f(x)\ e^{-ix\xi}\ dx$ is a subset of the fundamental interval $[-\pi, +\pi]$. Schoenberg, in his seminal 1946 papers has introduced his operators

$$T_\phi\ :\ f \mapsto \sum_{\alpha \in \mathbf{Z}} f(\alpha)\ \phi(x - \alpha)$$

by replacing the sinc function by a cardinal B-spline in order to have local operators which can be managed quite efficiently, and these operators show a wealth of useful applications in Computer Aided Geometric Design.

In this talk we will report on recent advances in a more general setting by looking at a set $\mathbf{\Phi} = \{\phi_1, \ldots, \phi_n\}$ of compactly supported $L_2(\mathbf{R}^d)$-functions, and at a corresponding polyphase sampling operator

$$T_{\mathbf{\Phi}}\ :\ f \mapsto \sum_{\alpha \in \mathbf{Z}} \sum_{p=1}^{n} f(\alpha + a_p)\ \phi_p(x - \alpha)\ ,$$

for given (different) shifts $a_p \in [0, 1[$, $p = 1, \ldots, n$. In particular, we will deal with the following questions and properties of these operators:

- Accuracy, *i.e.*, order of polynomial reproduction.
- Approximation order of the scaled operators, and
- Asymptotic error expansions.

We will also show the connection to the notion of balancing which was recently introduced by Lebrun and Vetterli, and others.