Mathematics Colloquium at IUB

GRIGORY LITVINOV

(IUB and Independent University of Moscow)

will speak on

Dequantization of Mathematics

Date: **Monday, October 17, 2005**
Time: **17:15**
Place: **Lecture Hall Research II, IUB**

Abstract:

The traditional mathematics over numerical fields can be dequantized as the Planck constant $\hbar$ tends to zero taking imaginary values. This dequantization leads to the so-called idempotent mathematics based on replacing the usual arithmetic operations by a new set of basic operations (e.g., such as maximum or minimum), that is on the concepts of idempotent semifield and semiring.

Typical (and the most common) examples are given by the so-called $(\max, +)$ algebra $\mathbf{R}_{\max}$ and $(\min, +)$ algebra $\mathbf{R}_{\min}$. Let $\mathbf{R}$ be the field of real numbers. Then $\mathbf{R}_{\max} = \mathbf{R} \cup \{-\infty\}$ with operations $x \oplus y = \max\{x, y\}$ and $x \odot y = x + y$. Similarly $\mathbf{R}_{\min} = \mathbf{R} \cup \{+\infty\}$ with the operations $\oplus = \min$, $\odot = +$. The new addition $\oplus$ is idempotent, i.e., $x \oplus x = x$ for all elements x. Of course, $\mathbf{R}_{\max}$ and $\mathbf{R}_{\min}$ are isomorphic idempotent semifields.

There exists a correspondence between interesting, useful and important constructions and results in the traditional mathematics and similar constructions and results in idempotent mathematics. This heuristic correspondence can be formulated in the spirit of the well-known N. Bohr's correspondence principle in quantum mechanics; in fact, the two principles are intimately connected. For example, the Hamilton–Jacobi equation is an idempotent version of the Schrödinger equation, the variational principles of classical mechanics can be treated as an idempotent version of the Feynman path integral approach to quantum mechanics. The Legendre transform turns out to be an idempotent version of the Fourier transform etc. A systematic and consistent application of the idempotent correspondence principle leads to a variety of results (often quite unexpected) in different areas including algebra, geometry, mathematical physics, differential equations, optimization, analysis and numerical analysis, stochastic problems, computer applications.

Colloquium Tea at ca. 16:45 in the Tea Room of Research II, close to the lecture hall. Everybody is welcome!

M. STOLL