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Congruences between modular forms of genus 1 and of genus 2

Modular forms f of any genus, which are eigenforms for the Hecke algebras, produce sequences of numbers (algebraic integers, sometimes rational integers) indexed by the primes

$$\{\lambda_p(f)\}_{p\in \text{ set of primenumbers}}.$$

Two classical examples are given by the Δ function and the Eisenstein series

$$\Delta(q) = q\prod(1-q^n)^{24} = q - 24q^2 + 252q^3 - 1472q^4 + 4830q^5 + \ldots = \sum \tau(n)q^n$$

$$E_{12}(q) = \frac{691}{65520} + q^1 + (1+2^{11})q^2 + (1+3^{11})q^3 + (1+2^{11}+4^{11})q^4 + (1+5^{11})q^5 \ldots.$$

Ramanujan discovered the famous congruences: For all primes p we have

$$\tau(p) \equiv 1 + p^{11} \bmod 691$$

(Example: $4830 \equiv 1 + 5^{11} \bmod 691$.)

In my lecture I will discuss some conjectures, which predict generalizations of these congruence to congruences between modular forms of genus 1 and genus 2. I have very strong theoretical reasons for the validity of these congruences and they have been confirmed by numerical data.